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Newtonian Gravitational Theory: Interaction with Light

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Abstract

The interaction of light with the gravitational field of a mass point described by a Newtonian gravitational field theory gives the same gravitational red shift as accepted theory. The dual force which is an integral part of the classical field theory and which has been shown to give the same advance of the perihelion of the orbit as Einstein's General Theory of Relativity is also the reason that light is deflected in the neighborhood of a massive particle. The deflection predicted is slightly more than 10% larger than Einstein's value, but within the experimental error of observational data. The dual force and its effects must be taken seriously. Its role in electrodynamics and quantum mechanics is briefly discussed.

1. *Introduction*

Newtonian Gravitational Field Theory (Schwebel, 1970) reveals that in addition to a counterpart of the Lorentz force of electromagnetic origin there is a dual force. The inclusion of the dual force in the equations of motion for two bodies led to a number of interesting results (Schwebel, 1971). Among these was the advance of the perihelion of the orbit of one particle about the other which agreed with the result obtained from Einstein's General Theory of Relativity. The present paper extends the application of the theory to the motion of light in the gravitational field of a mass particle.

We will find that the theory yields the correct gravitational red shift and, within experimental error, the observed deflection of light in a gravitational field. The former result is independent of the dual force, whereas the latter is not. The value obtained for the deflection of light is slightly more than ten percent larger than Einstein's.

We will proceed from the set of equations and relations obtained from the application of the theory to the two-body problem (Schwebel, 1971). These relations will be altered so that we can identify one of the bodies as a photon and take into account the interaction between light and matter. Once the pertinent equations have been established, they will be solved exactly. Finally, we will discuss the results and their significance, especially, for electrodynamics and quantum mechanics.

2. *Equations of Motion*

In the notation of an earlier article (Schwebel, 1971), the equations that describe the interaction between two massive bodies are: (1) The conservation of energy relation,

$$p_2{}^0 + \frac{\mu_1 \mu_2}{r} = E = \text{const.}; \qquad \mu = \sqrt{(-G)}\, m \tag{2.1}$$

The conservation of angular momentum equation,

$$\mathbf{r} \times \mathbf{p}_2 - \lambda\mu_1 \mu_2 \mathbf{r}/r = \mathbf{L} = \text{const.} \tag{2.2}$$

and the trajectory of one particle relative to the other,

$$\frac{1}{r} = A[1 + B \sin\beta(\phi - \phi_0)] \tag{2.3a}$$

$$\cos\theta = -\lambda\mu_1 \mu_2/L \tag{2.3b}$$

where the subscripts 1 and 2 refer to the two massive particles; λ is a constant ($\lambda^2 = 5$), which is a measure of the strength of the dual force relative to the conventional Newtonian gravitational force; $\mathbf{p}_2$ and $p_2{}^0$ are the components of the four-momentum of particle two; A, B and β are as follows:

$$A = (\mu_1 \mu_2 E)/(\mu_1{}^2 \mu_2{}^2 - L^2 \sin^2\theta)$$
$$B = [m_2{}^2 + \lambda^2 \tan^2\theta (E^2 - m_2{}^2)]^{1/2}/E$$
$$\beta = [\sin^2\theta - (\cos^2\theta/\lambda^2)]^{1/2}$$

and $L = |\mathbf{L}|$.

We identify particle two with the photon. This requires that, in the above equations, we set

$$|\mathbf{p}_2| = h\nu, \qquad p_2{}^0 = h\nu\ (c = 1), \qquad m_2 = 0$$

and, for the photon, we must have

$$p^2 - p^{02} = 0 \tag{2.4}$$

In what follows, the subscript two will be dropped in order to indicate that the massive particle two has been replaced by a photon.

3. *Gravitational Red Shift*

Performing the indicated changes on equation (2.1) we find that it reduces to

$$h\nu - (Gm_1/r)m = h\nu + \Phi m = E \tag{3.1}$$

where $\Phi \equiv (-Gm_1/r)$ is the gravitational potential due to the massive particle. The m which appears in the equation is the effective mass of the photon determined from the relation $E = m(c = 1)$.

Equation (3.1) is a well-known starting point for the derivation of the gravitational red shift (Adler *et al.*, 1965). Note that the dual force plays no role in the determination of the red shift. All that is required is the conservation of energy and the properties of the photon.

4. *Deflection of Light*

The dual force does play an all important role in the deflection of light in a gravitational field. To see this, let us introduce a rectangular coordinate system with its z-axis parallel to $\mathbf{L}$. Equation (2.2) can be expressed in component form in such a coordinate system and solved. The results of the straightforward algebraic calculations are:

$$\mathrm{p}_x = \frac{-\kappa y}{zr} + \frac{x}{z}\mathrm{p}_z, \tag{4.1a}$$

$$\mathrm{p}_y = \frac{\kappa x}{zr} + \frac{y}{z}\mathrm{p}_z \tag{4.1b}$$

and

$$z/r = \kappa/L \tag{4.1c}$$

where $\kappa \equiv \lambda G m_1 m$ and $L = |\mathbf{L}|$. The last equation can also be obtained by taking the scalar product of equation (2.2) with $\mathbf{r}$. Then equation (4.1c) takes the form $\cos\theta = \kappa/L$ which is identical to equation (2.3b). The angle θ is between the radius vector $\mathbf{r}$ and the z-axis. Equation (4.1c) describes the trajectory of the photon as lying on the surface of a cone whose vertex is occupied by the massive particle and whose axis is parallel to $\mathbf{L}$. The semi-angle of the cone is θ.

From equations (4.1a, b, c) we obtain

$$p^2 = \frac{L^2 - \kappa^2}{r^2} + \frac{L^2}{\kappa^2}\mathrm{p}_z^{\,2} \tag{4.2}$$

Using equations (2.1), (2.4) and (4.2), we find that

$$p_z^{\,2} = \frac{\kappa^2}{L^2 r^2}[(Er + Gm_1 m)^2 - (L^2 - \kappa^2)] \tag{4.3}$$

It follows from equation (4.2) that for $r \to \infty$ we have

$$\lim(\mathrm{p}_z/\mathrm{p}) = \pm\frac{\kappa}{L} = \pm\cos\theta \qquad r \to \infty \tag{4.4}$$

Therefore the vector p—the momentum of the photon—must lie along a generator of the cone, the radius vector, when incident from infinity or receding to a great distance relative to the particle at the apex. In other words, the photon on incidence is directed towards the apex of the cone and on emergence, i.e., at a great distance from the massive particle after interacting with it, is directed away from the apex along the radius vector.

If there is no azimuthal component (and we shall show that there is such a deflection but it is negligible) then the total deflection of the light is $\pi - 2\theta$.

If we let $\alpha = (\pi/2) - \theta$, it follows from equation (4.1c) in which $(z/r) = \cos\theta$ and $\kappa/L \ll 1$ that $\alpha = \kappa/L$ and that the total deflection is

$$2\alpha = (2\lambda G m_1 m)/L$$

To evaluate 2α we must determine L. From equation (2.2), we see that

$$\begin{aligned} |\mathbf{r} \times \mathbf{p}| &= \{L^2 - \kappa^2\}^{1/2} \\ &\sim L \end{aligned}$$

Since $\kappa/L \ll 1$. The closest distance of approach, $r(\text{min})$, of the photon to the massive particle occurs when $p_z = 0$. At that point $\mathbf{r}$ is perpendicular to p, the momentum of the photon, and can be calculated from equation (4.3). It follows that for a photon $L = r(\text{min}) \cdot h\nu$. With this value for L and with $m = h\nu$ we find that

$$2\alpha = 2\surd(5) \frac{G m_1}{r(\text{min})} \tag{4.5}$$

Einstein's result for the same quantity is $4Gm_1/r(\text{min})$. The ratio of these values is $2\sqrt{5}/4 = 1{\cdot}118$.

The azimuthal contribution to the deflection of the light can be calculated from equation (2.3a). When r becomes infinite, we see that

$$\sin\beta(\phi - \phi_0) = -\frac{1}{B}$$

For the photon $B = \lambda \tan\theta = \sin\theta/(Gm_1 m/L)$. We have seen that $Gm_1 m/L$ is a very small quantity. Therefore, it follows that

$$\begin{aligned} (\phi - \phi_0) &\sim -\frac{1}{\beta B} = -\frac{G m_1 m}{\beta \sin\theta L} \\ &\frac{\sim G m_1 m}{L} \end{aligned}$$

since $\beta \sim 1$ and $\sin\theta \sim 1$.

In the absence of the dual force the trajectory of the photon lies in a plane, and its deflection would be given by $2(\phi - \phi_0) = 2Gm_1 m/L$ which reduces to $2(\phi - \phi_0) = 2Gm_1/r(\text{min})$. This result is the one usually obtained by conventional classical approaches to this problem (Kittel *et al.*, 1965). The present theory adds the complexity that the trajectory of the photon lies on the surface of a cone and not in a plane. The deflection of light, as we have stated above, would be $\pi - 2\theta$ if there were no deflection in the azimuthal plane. Due to the azimuthal deflection there is a change in the angle of deflection of the second order in $(\phi - \phi_0)$ which is negligible. The exact relation between the semi-angle of the cone, θ, and the semi-angle, ω, between the radial vectors to the incident and emerging direction of the photon is

$$\sin\omega = \sin\theta \cos(\phi - \phi_0) \tag{4.6}$$

Because $\phi - \phi_0$ is small, it follows that ω and θ are equal at least to the second order in $\phi - \phi_0$. If $\theta = \pi/2$, i.e., the dual force is not taken into account, then $\omega = (\pi/2) - (\phi - \phi_0)$ and the deflection of the light occurs in the surface of a plane and is equal to $\pi - 2\omega = 2(\phi - \phi_0)$. A result which we pointed out before is obtained in most classical solutions to the problem.

5. *Discussion*

An earlier study (Schwebel, 1971) showed that the inclusion of the dual force in the equations of motion predicted the same analytical result for the advance of the perihelion of the orbit of one particle gravitating about a second particle as Einstein obtained from the General Theory of Relativity. The present analysis shows that the same dual force is responsible for the observed deflection of light in a gravitational field.

The significance of the above results is not only that they demonstrate that significant experimental data are predicted, but also that an integral part of a classical field theory, the dual force, accounts for the observations. We conclude that the dual force must be taken seriously; it must be included in all theoretical analyses of phenomena by electromagnetic theory and quantum theory.

More specifically, we have pointed out (Schwebel, 1970) that a dual electromagnetic force exists. Among the equations of motion which describe the electromagnetic interactions between two charged particles, we find an equation analogous to equation (2.2). Thus we will find an intrinsic angular momentum of constant magnitude as a consequence of incorporating the dual force into the equations of motion. The intrinsic angular momentum is proportional to e^2/c or to $\alpha\hbar$ where α is the fine structure constant.

The last conclusion shows the importance of exploring the effects of the dual force in the framework of quantum theory. The transcription of equation (2.2) into quantum mechanical terms seems to hold the promise of explaining the anomalous magnetic moment of the electron-proton system as due to the heretofore neglected dual force. Work that has been done along these lines will be submitted for publication in the near future.

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